Problem Set 3: The Earth's Gravity

EAS 3610/8803: Introduction to Geophysics

Assigned:	10/09/20 (Fri.)
Due:	10/16/20 (Fri.)

Name: ______________________________________

Others consulted: ________________________________

A note about your homework: Please be neat and organized! Once you have found a way to the answer, please rewrite it in an orderly fashion so that others can follow your steps, and put a box around your final solution, when appropriate. Include this page as the cover, show all of your work, and list all who helped with this set, including your instructors. Relative problem values are shown in [] at the beginning of the problem.

Overcoming gravity

1. [**40**] Like the moon, artificial satellites are 'held' in the sky by a balence between the gravitational force F_g exerted by the earth, representing a centripetal force, and a centrifugal pseudoforce F_c pulling it away due to its rotation about the Earth (caused by the linear acceleration that is tangential to the rotational path). We've learned that:

$$F_g = \frac{Gm_1m_2}{r^2} \tag{1}$$

where G is the Universal Gravitational Constant $= 6.674\times10^{-11}\text{N m}^2\text{ kg}^{-2}$, r is the distance between the satellite of mass m_1 and the body its orbiting of mass m_2.

We can write the centrifugal force as:

$$F_c = m_1 r\omega^2 \tag{2}$$

where ω the instantaneous angular velocity, which could be considered constant for a near circular orbit and related to the orbital period T, such that:

$$T = 2\pi/\omega \tag{3}$$

In Newtonion mechanics the gravitational force F_g exerted by the earth, representing a centripetal force, directly counters this centrifugal force F_c, 'holding satellites in the sky'.

$$F_g = F_c. \tag{4}$$

   (a) Write out the above force-balance equation (4) in terms of T.

   (b) How does the period of a satellite's orbit depend on its own mass?

   (c) Now, determine the altitude (distance above the surface of the earth) of a satellite in geosynchronous orbit (T = 23.93447 hours = 1 sidereal day).

   (d) The International Space Station (ISS) is in low-earth orbit, traveling only approximately 350 km above the surface of the earth. How fast is its orbital period (note its altitude above the earth's surface is not its orbital radius)?

Gravity of the Earth's interior

2. **[40]** Using the knowledge that within a spherical body, gravitational acceleration does not account for material that is of a greater radial distance than the point of measure:

   (a) Determine $g(r)$, from the surface to the center of the earth, with measurements at every 1000 km interval. Make additional measurements as necessary at each, the top of the mantle, the core-mantle boundary, the outer-core/inner-core boundary, and the center of the earth. Note, you may want to use some of your methods and results from problem set 1.

   (b) Plot these results on an X-Y plot with g on the X-axis and Y from the surface ($r = 6371$ km) to the center of the Earth ($r = 0$).

   (c) Discuss anything that you find particularly interesting along this profile. Are variations in gravity with depth a major concern for studies of mantle dynamics?

Gravitational Corrections and Isostasy

3. **[40]** Given the below figure of a local airborne gravity survey:

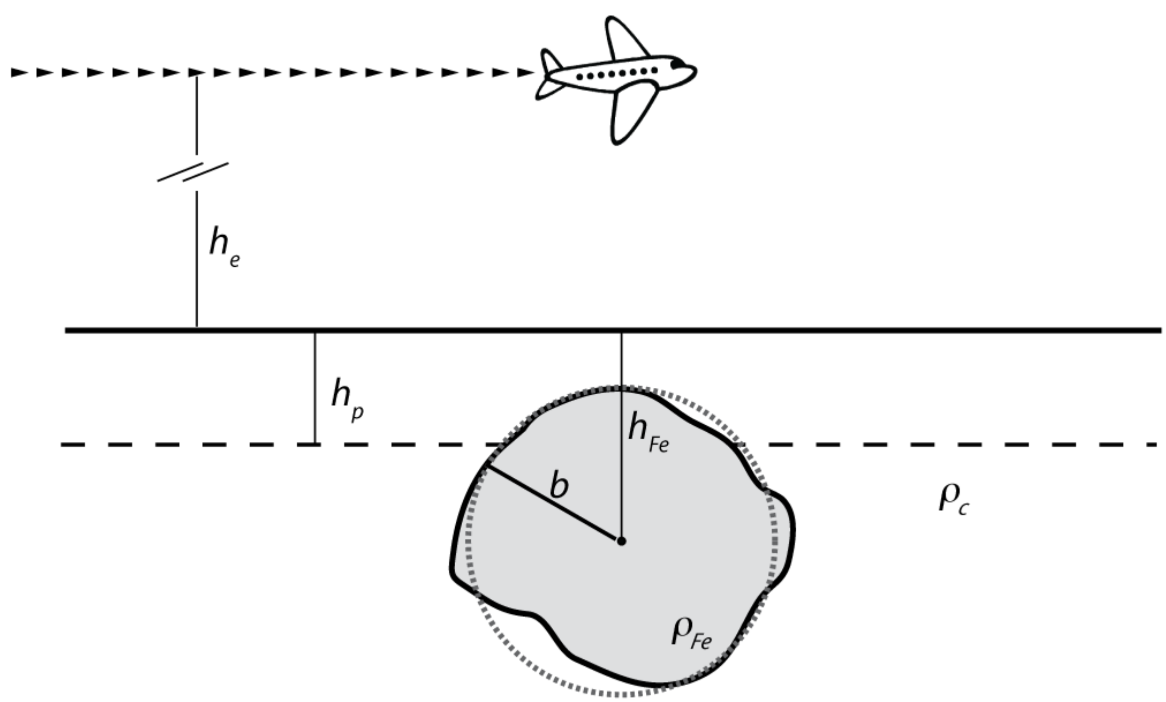

Figure 1: An aircraft traveling over a extremely flat plateau of height $h_p = 1$ km above the reference ellipsoid. However, buried within this crust is a large relatively spherical massive hematite (Fe_2O_3) deposit, the primary ore material mined for iron. The hematite, which has density $\rho_{Fe} = 5300$ kg m^{-3}, is centered about 3 km below the surface, and has about 2 km radius, b. The aircraft is flying at an elevation $h_e = 300$ m above the ground, and is surveying land near Atlanta at $\lambda = 34°$ N (BTW, I'd be really surprised to find such a deposit around Atlanta).

   (a) With proper assumptions of shallow crustal density, what would should the downward component of the gravity field be if it were flying over a section of the observed plateau with no gravitational anomaly (no hematite deposit)?

   (b) Now, how would be the measured anomaly (difference from a base model) as the plane flies directly over the hematite body (as shown)?

   (c) How would this value change if the iron ore were completely isostatically compensated by a crustal root in the mantle?

Graduate Section Homework

Final Presentation:

During the last classes in Geophysics (11/18, 11/20) you will present on a topic of geophysical research that either you are currently working on, or is of particular interest to you. The presentation will be done in the standard *American Geophysical Union* meeting format, of 12 minutes of presentation, with 3 minutes of question and answer.

With this homework set, submit a 1-2 sentence description of the topic that you choose to present. If you are having trouble deciding, please discuss this with me. Later on, you will be asked to submit an abstract (one paragraph summary) of your topic.