Problem Set 1: Mathematical Approximation and Plate Motions

EAS 3610/8803: Introduction to Geophysics

Assigned: 9/04/20
Due: 9/11/20

Name: ______________________________

Others consulted: ______________________________

- Please be neat and organized! Once you have found a way to the answer, please rewrite it in an orderly fashion so that others can follow your steps, and put a box around your final solution when appropriate.
- Include this page as the cover, listing all who helped with this set including me in the "Others consulted" line.
- Show all of your work.
- An answer with incorrect or absent units will be considered wrong.
- For electronic submission to Canvas, please create a single PDF of your written work. This can be done with most smartphones using tools such as *Google Drive*, *Adobe Scan*, or *Scanner App* (I do not personally endorse any of these tools, and have only tried GoogleDrive).

Mathematical Approximation

1. Frequently, in Earth Science research we must make calculations based on physical properties that are either not precisely known or are very difficult to directly measure. This, of course, requires us to make scientifically justifiable approximations that will allow us to get closer to understanding the forces behind the processes. Frequently, Earth Scientists, particularly Geophysicists are delighted to get within a factor of two (sometimes an order of magnitude). Such is the case in the title example of the book *Consider a Spherical Cow* by John Harte. To illustrate this example, we will do just that.

   (a) To start, estimate the approximate mass of the cow, by simply assuming it to be a single sphere of reasonable radius and density. Use your own logic to determine these values, and explain how you came to these decisions.

   (b) Now, assume that the cow is 10% efficient at turning food into muscle, how much grass and similar would the creature eat over its growing life?

2. Now, lets apply the same principles of approximation to the Earth. Let us consider calculating the mass of a spherical Earth, but do it in a couple of different ways.

   (a) 1^{st} *order estimate*: Calculate the Earth's mass using the **a singular approximate average whole-earth density** obtained by visually examining the PREM density model (Fig. 8.5 and Appendix 9 in *The Solid Earth* by CMR Fowler).

   (b) 2^{nd} *order estimate*: Now, using the same information from the PREM model **approximate three average densities**: inner core, outer core, and mantle with crust (the crust really is insignificant). Use this information to determine the mass of the respective layers/shells, sum them up to find your new mass of the Earth. You will need to use the equation for the volume, V, of a thick shell with outer and inner radii (r_o, r_i):

$$V = \frac{4}{3}\pi(r_o^3 - r_i^3). \tag{1}$$

(c) Finally, look up a published value for the mass of the Earth, identifying the reference. Discuss the difference between your estimates and the published value for the mass of the Earth. Do you feel your approximations were good first and second order estimates? What do you feel was your primary source of error in each case?

Plate Kinematics

Plate Kinematics is the study of the mechanics of plate motions without respect to the forces driving such motion. Here you will practice and develop a more full understanding of how plate motions work, how triple junctions evolve, and how plates move along the surface of a spherical Earth. These problems are in chapter 2 of *The Solid Earth* by Fowler.

3. **Problem 2.1:** Flat-Earth tectonic motions of a migrating triple-junction. Keep in mind that the rates shown on either side of the ridges are half-spreading rates, meaning they are the rate of one plate relative to the ridge. The plate rate of one spreading plate to another is twice (2×) that rate. This is not true for the convergence rate. Also, note that the RRR triple-junction is always stable and its position can be described relative to the half-rate of any one of the connected oceanic plates.

Graduate Section Homework

Euler's Fixed-Point Theorem

4. Write a routine using any programming language that you can [e.g. Python, Matlab, C++, FORTRAN, even Excel] to compute plate motion at any point on the surface of the Earth. The program should be such that given an Euler vector in the form (pole longitude, latitude, and rotation magnitude), you can determine the plate motion at any location (longitude, and latitude), in East and North velocities, V_E, V_N, as well as rate and direction, $|V|$, β.

   (a) Submit a printout of the routine.

   (b) For the sites listed in Table 1, calculate the relative present-day motions across each of these plate boundary sites. For each, plot the motion of the Pacific Plate relative to its abutting neighbor at that site. First, determine or calculate the proper Euler Pole vector to use. Recall that:

$$\vec{\omega}_{AC} = \vec{\omega}_{AB} + \vec{\omega}_{BC}, \tag{2}$$

   or

$$\vec{\omega}_{AC} = \vec{\omega}_{AB} - \vec{\omega}_{CB}. \tag{3}$$

   This is more complicated because per-component vector algebra only works when the coordinate system is orthogonal, which is not the case for the spherical geographic coordinates given in

Locale	Longitude [°]	Latitude [°]	Plate Motion local-relative
W Aleutian Trench	169	54	PA-NA
E Aleutian Trench	-169	52	PA-NA
San Andreas Fault	-122	38	PA-NA
Gulf of California	-110	26	PA-NA
East-Pacific Rise	-112	-13	PA-NZ
East-Pacific Rise	-110	-36	PA-NZ
Southern New Zealand	169	-45	PA-AU
Japan Trench	143	37	PA-EA

Table 1: Plate boundary sites Modified from Problem 2.3 in Fowler].

the table (long, lat, rotation). Thus, you must first convert the the components to cartesian coordinates, perform vector addition, and then revert back to spherical coordinates. [See Box example on page 23 of book.]. Usable relative Euler Poles are in Table 2.1 of *Fowler*.

(c) Plot the vectors on the attached map (Figure 1) or create your own. You will need to create a scale bar (vectors and scalebar are best created with a protractor and ruler (or completely with computer code).

(d) Anything of particular interest to note at the Japan Trench site? Are you aware of any particular geologic/geophysical events of note in this area that you may want to discuss in association with what you've measured?

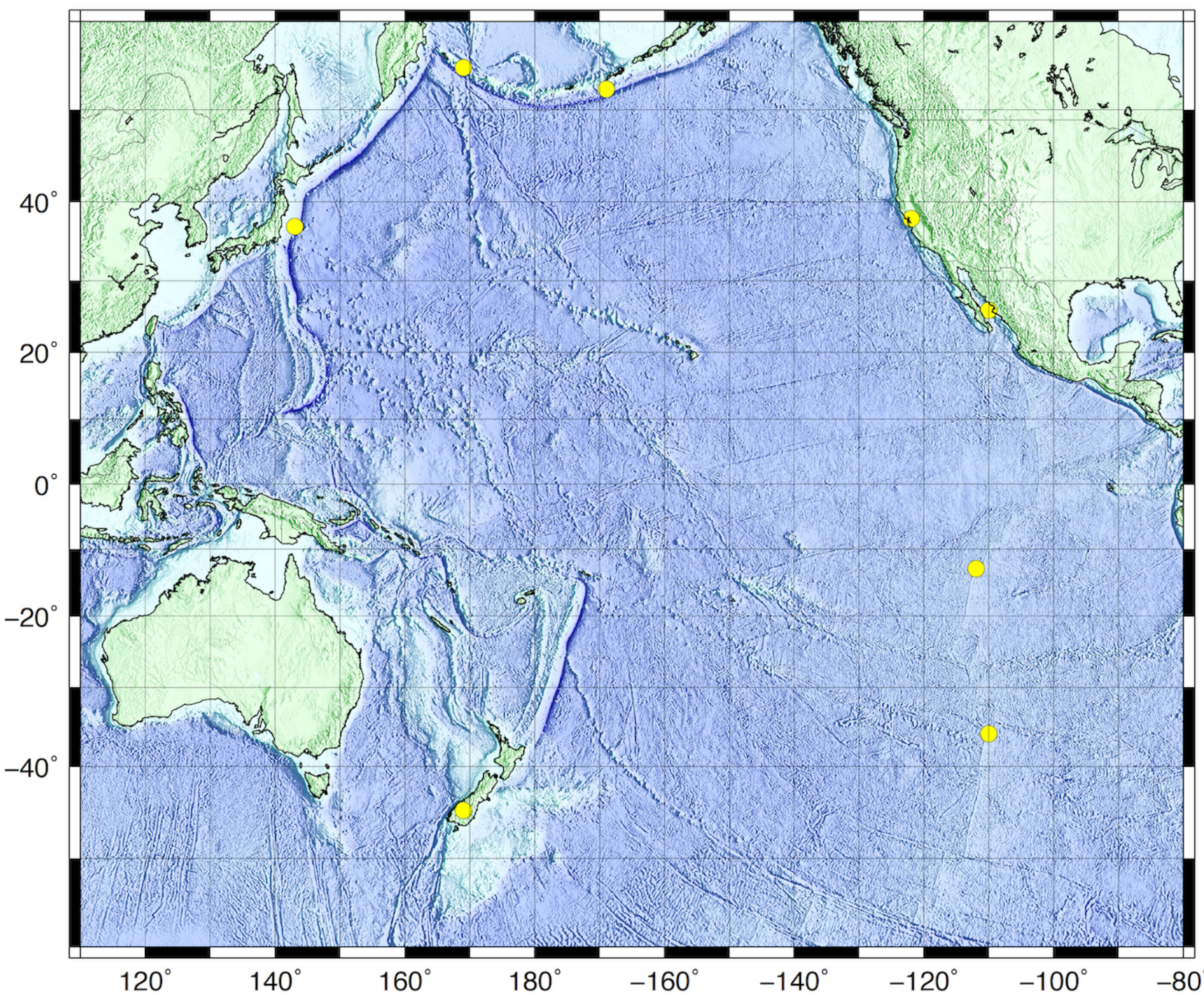

Figure 1: Physiographic map of the Pacific and surrounding area. Yellow points are sites in Table 1.